STAT 2593

Lecture 028 - Confidence Intervals for the Variance and Standard Deviation of a Normal Population

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Confidence Intervals for the Variance and Standard Deviation of a Normal Population

Learning Objectives

1. Construct confidence intervals for the variance and standard deviation in normal populations.

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  - Recall that the χ^2_ν distribution is a gamma family distribution!
  - This allows us to use the *same procedures* as before, with χ^2 critical values.

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Summary

- The sample variance has a χ^2_ν sampling distribution.
- The degrees of freedom are given by $\nu = n - 1$.
- The sampling distribution can be inverted to obtain confidence intervals for the variance and standard deviation.